51. If matrix \( A = \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix} \) then \( A^3 \) is given by

(A) \( \begin{pmatrix} \cos 3x & \sin 3x \\ -\sin 3x & \cos 3x \end{pmatrix} \)

(B) \( \begin{pmatrix} \cos 2x & \sin 3x \\ -\sin 2x & \cos 3x \end{pmatrix} \)

(C) \( \begin{pmatrix} \cos 2x & \sin 3x \\ -\sin 3x & \cos 2x \end{pmatrix} \)

(D) \( \begin{pmatrix} \cos 3x & -\sin 3x \\ \sin 3x & -\cos 3x \end{pmatrix} \)

52. For what values of \( \alpha \), the rank of matrix

\[
A = \begin{bmatrix}
1 & 1 & -1 & 0 \\
4 & 4 & -3 & 1 \\
\alpha & 2 & 2 & 2 \\
9 & 9 & \alpha & 3
\end{bmatrix}
\]

is 3.

(A) \( \alpha = 6, -2 \)

(B) \( \alpha = -6, 2 \)

(C) \( \alpha = -3, 2 \)

(D) None of these

53. The approx. positive root of the equation:

\[ x^3 - 12.2 \ x^2 + 7.45 \ x + 42 = 0 \]

lying between \( x = 11 \) and \( x = 12 \) by Regula Falsi method is given by

(A) 11.4994

(B) 11.1194

(C) 11.1994

(D) 11.9994

54. The matrix obtained by:

\[
\begin{pmatrix}
1 & -\tan x \\
tan x & 1
\end{pmatrix}
\]

is given by

(A) \( \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix} \)

(B) \( \begin{pmatrix} \cos 2x & \sin 2x \\ -\sin 2x & \cos 2x \end{pmatrix} \)

(C) \( \begin{pmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{pmatrix} \)

(D) \( \begin{pmatrix} \cos 3x & -\sin 3x \\ \sin 3x & -\cos 3x \end{pmatrix} \)
55. Using Runge Kutta (fourth order) method, where \( \frac{dy}{dx} = x + y \) and \( y(0) = 1 \), approximation to \( y(0.1) \) correct to five decimal places in steps of \( h = 0.1 \) is given by option:

(A) 2.11133  
(B) 1.11034  
(C) 1.21135  
(D) 1.23230

56. Determine \( \lambda \) and \( \mu \) such that the equations

\[
\begin{align*}
x + y + z &= 6, \\
x + 2y + 3z &= 10, \\
x + 2y + \lambda z &= \mu
\end{align*}
\]

have infinite number of solutions.

(A) \( \lambda = 3, \mu \neq 10 \)  
(B) \( \lambda = -3, \mu = -10 \)  
(C) \( \lambda = 3, \mu = 10 \)  
(D) \( \lambda \neq 3 \) and \( \mu \) can have any value

57. If \( \phi(x, y, z) = 3x^2y - y^3z^2 \), value of grad. \( \phi \) at the point \((1, -2, -1)\) is given by

(A) \(-12i + 9j - 16k\)  
(B) \(12i - 9j - 6k\)  
(C) \(-12i - 9j - 16k\)  
(D) None of these

58. The rank of matrix

\[
A = \begin{bmatrix}
5 & 6 & 7 & 8 \\
6 & 7 & 8 & 9 \\
11 & 12 & 13 & 14 \\
16 & 17 & 18 & 19
\end{bmatrix}
\]

is given by

(A) 2  
(B) 3  
(C) 4  
(D) None of these

59. The \( n^{th} \) derivative of the function \( y = x^4/(x-1)(x-2) \) is

(A) \((-1)^n n! \left[ \frac{16}{(x-2)^{n+1}} - \frac{1}{(x-1)^{n+1}} \right] (n > 2)\)  
(B) \(n! \left[ \frac{16}{(x-2)^{n+1}} - \frac{1}{(x-1)^{n+1}} \right] (n > 0)\)  
(C) \((-1)^n \left[ \frac{16}{(x-2)^{n+1}} - \frac{1}{(x-1)^{n+1}} \right] \)  
(D) \((-1)^n+1 n! \left[ \frac{16}{(x-2)^{n+1}} - \frac{1}{(x-1)^{n+1}} \right] (n > 2)\)
60. For the matrix \( A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix} \), find the matrix represented by \( A^8 - 5A^7 + 7A^6 - 3A^5 - 5A^3 + 8A^2 - 2A + I \).

(A) \( \begin{pmatrix} 8 & 4 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 9 \end{pmatrix} \)

(B) \( \begin{pmatrix} 8 & 5 & 1 \\ 6 & 3 & 0 \\ 5 & 5 & 8 \end{pmatrix} \)

(C) \( \begin{pmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{pmatrix} \)

(D) None of these

62. The nature of an infinite series \( \sum \log \frac{n}{n+1} \) is said to be

(A) convergent

(B) absolute convergent

(C) divergent

(D) conditional convergent

63. A unit vector normal to the surface \( x^3 + y^3 + 3xyz = 3 \) at the point (1, 2, -1) is given by

(A) \( \frac{1}{\sqrt{11}} (-i + 3j - k) \)

(B) \( i - j + k \)

(C) \( \frac{1}{\sqrt{3}} (i + k) \)

(D) None of these

64. The eigen values of the following matrix

\[
\begin{bmatrix}
2 & -1 & 1 \\
-1 & 2 & -1 \\
1 & -1 & 2
\end{bmatrix}
\]

are given by solving the cubic equation

(A) \( \lambda^3 + 2\lambda^2 + 2\lambda - 13 = 0 \)

(B) \( \lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0 \)

(C) \( \lambda^3 + 7\lambda^2 + 2\lambda - 7 = 0 \)

(D) \( \lambda^3 - 6\lambda^2 + 12\lambda + 12 = 0 \)
65. If \( A = \begin{pmatrix} 1 & 2 \\ 5 & 7 \end{pmatrix} \), then \( A^{-1} \) is given by

(A) \( \frac{1}{3} \begin{pmatrix} -7 & 2 \\ 5 & -1 \end{pmatrix} \)

(B) \( \frac{1}{3} \begin{pmatrix} 7 & 2 \\ 5 & 1 \end{pmatrix} \)

(C) \( \begin{pmatrix} -7 & 2 \\ 5 & -1 \end{pmatrix} \)

(D) None of these

66. The quadratic form corresponding to the matrix \( A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 0 & 3 \\ 5 & 3 & 4 \end{bmatrix} \) is given by

(A) \( x_1^2 + 4x_2^2 + 4x_1x_2 + 10x_1x_3 + 3x_2x_3 \)

(B) \( x_1^2 + 4x_3^2 + 4x_1x_2 + 10x_2x_3 + 6x_2x_3 \)

(C) \( x_1^2 - 4x_3^2 + 4x_1x_2 + 10x_1x_3 + 6x_2x_3 \)

(D) \( x_1^2 + 4x_3^2 + 4x_1x_2 + 10x_1x_3 + 6x_2x_3 \)

67. The infinite series
\[
1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \frac{3.6.9.12}{7.10.13.16}x^4 + \ldots \infty
\]
is said to be convergent for

(A) \( |x| > 1 \)

(B) \( x < 1 \)

(C) \( |x| > -1 \)

(D) \( x < 2 \)

68. If \( y = \cos (m \sin^{-1} x) \), which is correct

(A) \( (1-x^2)y_{n+2} - (2n+1)xy_{n-1} \)

(B) \( (1-x^2)y_{n+2} - (2n+1)x_{n+1} \)

(C) \( (1-x^2)y_{n+2} - (2n+1)x_{n+1} \)

(D) \( (1-x^2)y_{n+2} - (2n+1)x_{n+1} \)

69. Given that \( f(u, v, w) \) is a differentiable function with \( u = x-y, \ v = y-z, \ w = z-x \), the value of \( \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} \) is given by

(A) 0

(B) 1

(C) −1

(D) None of these

70. For what value of \( a \) and \( b \), the following function is everywhere differentiable for the function given by
\[
f(x) = \begin{cases} 
  x^2 + 3x + a, & \text{for } x \leq 1 \\
  bx + 2, & \text{for } x > 1 
\end{cases}
\]

(A) \( a = 3, \ b = 4 \)

(B) \( a = 3, \ b = 5 \)

(C) \( a = 2, \ b = 5 \)

(D) \( a = -3, \ b = -5 \)
71. For the functions \( f(x) = x^3 + 2 \) and \( g(x) = x^2 - 1 \) in the interval \([0, 1]\), the value of \( c \) for Cauchy's Mean Value theorem is given by

(A) \( c = 12/9 \)

(B) \( c = 14/6 \)

(C) \( c = 1/9 \)

(D) \( c = 14/9 \)

72. Using the expansion of \( \tan (x + h) \), the value of \( \tan 46^\circ \) correct to 4 significant figures is (where \( \pi = 3.14159 \))

(A) 0.5151

(B) 1.5051

(C) 1.1131

(D) 1.0355

73. The system

\[
\begin{align*}
x + 2y - 3z &= -1 \\
3x - y + 2z &= 7 \\
5x + 3y - 4z &= 2
\end{align*}
\]

is

(A) Inconsistent

(B) Consistent with trivial solution

(C) Consistent with unique solution

(D) Consistent with more than one solution

74. The expansion of \( f(x, y) = e^x \cos y \) about the point \((0, 0)\) is given by

(A) \( 1 - x + \frac{1}{2} [x^2 + y^2] + \frac{1}{6} [x^3 - 3y^2x] + ... \)

(B) \( 1 + x + \frac{1}{2} [x^2 - y^2] + \frac{1}{6} [x^3 - 3y^2x] + ... \)

(C) \( 1 + x - \frac{1}{2} [x^2 - y^2] + \frac{1}{9} [x^3 - 3y^2x] + ... \)

(D) None of these

75. The main difference between Jacobi's and Gauss-Seidal Method is

(A) Deviation from the correct answer is more in Gauss-Seidal

(B) Convergence in Jacobi's method is faster

(C) Computations in Jacobi's can be done in parallel but not in Gauss-Seidal

(D) Gauss-Seidal can solve the system of linear equations in three variables whereas Jacobi cannot

76. Given that \( u = \tan^{-1} \left( \frac{x^3 + y^3}{x + y} \right) \), the value of \( x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} \) is given by

(A) \( \sin u \)

(B) \( \sin 2u \)

(C) \( \sin 3u \)

(D) \( \tan u \)
77. Value of \( \int_0^{2\pi} \frac{1}{2 + \cos \theta} \, d\theta \) is given by the option:
(A) \( \frac{2\pi}{3} \)
(B) \( \frac{\pi}{2} \)
(C) \( \frac{2\pi}{\sqrt{3}} \)
(D) \( \frac{3\pi}{\sqrt{2}} \)

78. If \( \frac{dy}{dx} = -\frac{F_x}{F_y} \) and \( \frac{d^2y}{dx^2} = -(F_{xx} F_y^2 - p F_x F_y) F_y \), given that the equation \( F(x, y) = 0 \) defines \( y \) implicitly as a differentiable function of the independent variable \( x \) and \( F_y \neq 0 \), then value of \( p \) is given by:
(A) 3
(B) 2
(C) -2
(D) 1

79. The solution of \((y^3 + 2y)dx + (x^3 + 2y^4 - 4x)dy = 0\) is given by:
(A) \( x\left(y + \frac{2}{y^2}\right) + y^3 = c \)
(B) \( x\left(y + \frac{2}{y^2}\right) + y^2 = c \)
(C) \( x\left(y + \frac{1}{y^2}\right) + y^2 = c \)
(D) \( y\left(y + \frac{2}{y^2}\right) + y^2 = c \)

80. The residue at each pole of the function \( f(z) = \cot z \) is given by:
(A) 4
(B) 2
(C) 0
(D) 1

81. The Particular Integral of the equation:
\[(D^3 - 7D^2 + 10D)y = e^{2x} \sin x\]
is given by:
(A) \( \frac{e^x}{50} (7 \cos x - \sin x) \)
(B) \( \frac{e^{2x}}{50} (7 \cos x - \sin x) \)
(C) \( \frac{e^{2x}}{50} (7 \cos x + \sin x) \)
(D) \( \frac{e^{2x}}{5} (7 \cos x + \sin x) \)

82. The value of \( \int_C \frac{1}{z^2 - 1} \, dz \), where \( C \) is the circle \( |z| = 2 \) is:
(A) 0
(B) 2
(C) 3/4
(D) 2
83. If \( \phi(x, y) = \frac{x}{x^2 + y^2} \), the magnitude of the directional derivative along a line making an angle \( 30^\circ \) with the positive direction of x-axis at point \((0, 2)\) is given by

(A) \( \frac{\sqrt{5}}{8} \)

(B) \( \frac{1}{\sqrt{2}} (i + k) \)

(C) \( \frac{\sqrt{3}}{8} \)

(D) \( \frac{\sqrt{3}}{8} i \)

84. In which option, algebraic structure is not semi group

(A) \((N, +)\)

(B) \((Z, −)\)

(C) \((N, +), (Z, −)\)

(D) None of these

85. The mobius transformation which maps the points \( z = −i, 0, i \) into the points \( w = −1, i, 1 \) respectively is given by

(A) \( w = \frac{-iz + i}{z + 2} \)

(B) \( w = \frac{iz + i}{z + 1} \)

(C) \( w = \frac{-iz + i}{z + 1} \)

(D) \( w = \frac{-iz + i}{z - 1} \)

86. The \( n^{th} \) derivative of \( y = x^{n−1} \log x \) is given by

(A) \( \frac{n!}{x} \)

(B) \( \frac{(n+1)!}{x} \)

(C) \( \frac{(n−1)!}{x} \)

(D) \( \frac{(n−1)!}{2x} \)

87. By method of variation of parameters, the solution of equation \( y'' + y = \cosec x \) is given by

(A) \( A \cos x + B \sin x - (x \sec x + \log \tan x) \)

(B) \( A \cos x + B \sin x + x \cos x + \sin x \log \sin x \)

(C) \( A \cos x - B \sin x - \cos x \log (\sec x \cdot \tan x) \)

(D) \( A \cos x + B \sin x - x \cos x + \sin x \log \sin x \)

88. Using Green's theorem, value of

\[ \int_c (x^2 + xy)dx + (x^2 + y^2)dy \]

where \( c \) is the square formed by the lines \( y = ± 1, x = ± 1 \).

(A) \( \frac{3}{8} \)

(B) \( \frac{5}{12} \)

(C) 1

(D) 0
89. The solution of equation by method of separation of variables, where \( \frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u \) and given that \( u(x, 0) = 6e^{-3x} \) is given by
(A) \( u = 6e^{-3x-3y} \)
(B) \( u = 6e^{3x} + 2y \)
(C) \( u = 6e^{-3x-2t} \)
(D) \( u = 6e^{-3x+2t} \)

90. The Particular integral of the differential equation:
\[
(D^2 + DD' - 6D^2)z = x^2 \sin (x + y)
\]
is given by
(A) \( \frac{1}{4} \left( x^2 + \frac{13}{8} \right) \sin (x + y) + \frac{3x}{8} \cos (x + y) \)
(B) \( \frac{1}{4} \left( x^2 - \frac{13}{6} \right) \sin (x + y) - \frac{13x}{8} \cos (x + y) \)
(C) \( \frac{1}{4} \left( x^2 - \frac{13}{8} \right) \sin (x + y) - \frac{3x}{8} \cos (x + y) \)
(D) \( \frac{1}{4} \left( x^2 - \frac{13}{8} \right) \cos (x + y) - \frac{13x}{8} \sin (x + y) \)

91. The Particular integral of the differential equation:
\[
(D^3 + 2D^2 + D)y = e^{2x} + x^2 + x
\]
is given by
(A) \( \frac{1}{18} e^{2x} + \frac{1}{3} x^3 - \frac{3}{2} x^2 + 4x \)
(B) \( \frac{1}{18} e^{2x} + \frac{5}{3} x^3 - \frac{3}{2} x^2 + 4x \)
(C) \( \frac{1}{18} e^{2x} + \frac{1}{3} x^3 - \frac{3}{2} x^2 + 3x \)
(D) \( \frac{1}{18} e^{2x} + \frac{1}{3} x^3 - \frac{3}{5} x^2 + 4x \)

92. A vector field \( F \) is given by \( F = (\sin y)i + x (1 + \cos y)j \), then value of integral \( \int_C F.d\bar{r} \)

Where \( C \) is the circular path given by \( x^2 + y^2 = a^2 \)
(A) \( 3 \pi a^2 \)
(B) \( 2 \pi a^2 \)
(C) \( \pi a^2 \)
(D) \( \frac{1}{2} \pi a^2 \)

93. The two regression equations of the variables \( x \) and \( y \) are \( x = 19.13 - 0.87y \) and
\( y = 11.64 - 0.50x \), the correlation coefficient between \( x \) and \( y \) is given by
(A) \(-0.26\)
(B) \(-0.36\)
(C) \(-0.66\)
(D) \(0.66\)

94. The Particular integral of \( (D^2 - D'^2)z = \cos (x + y) \) is given by
(A) \( \frac{x}{2} \cos (x + y) \)
(B) \( x \sin (x + y) \)
(C) \( 2x \sin (x + y) \)
(D) \( \frac{x}{2} \sin (x + y) \)
95. If \( z = xf(y/x) + g(y/x) \), then value of 
\[
\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2}
\] is given by

(A) \( \frac{-x}{y} \frac{\partial u}{\partial y} \)

(B) \(-1\)

(C) \(0\)

(D) None of these

96. The analytic function whose imaginary part is \( e^{-x} (x \sin y - y \cos y) \), is given by

(A) \( f(z) = e^{-z} (-z) + c \)

(B) \( f(z) = e^{-z} (z) + c \)

(C) \( f(z) = e^{z} (-z) + c \)

(D) \( f(z) = e^{-2z} (-z) + c \)

97. If the variance of the poisson distribution is 2 and given : \( e^{-2} = 0.1353 \), the value of \( P(r \geq 4) \) using recurrence relation of the poisson distribution is

(A) 0.1321

(B) 0.3613

(C) 0.1431

(D) None of these

98. A can hit a target 3 times in 5 shots, B 2 times in 5 shots and C 3 times in 4 shots. All of them fire one shot each simultaneously at a target. What is the probability that at least two shots hit the target?

(A) \( \frac{69}{100} \)

(B) \( \frac{9}{20} \)

(C) \( \frac{63}{100} \)

(D) \( \frac{18}{100} \)

99. The algebraic structure \( (Q, +, \cdot) \), \( (R, +, \cdot) \) represent

(A) Ring

(B) Field

(C) Group

(D) Commutative ring with unity

100. The solution of partial differential equations:
\[
x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)
\] is given by

(A) \( F(xyz, x^2 + y^2 + z^2) = 0 \)

(B) \( F(xyz, x^2 - y^2 - z^2) = 0 \)

(C) \( F(1/(xyz), x^2 + y^2 + z^2) = 0 \)

(D) None of these.
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### Answer Key: Maths

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